Geometry
Algebra Review: Simplifying Radicals

Name: Key
Date: _____ Period: _____

- *I can use properties of radicals to simplify radical expressions.*

An expression with radicals is in simplest form if the following are true:

- No perfect square factors other than 1 are in the radicand.
- No fractions are in the radicand.
- No radicals appear in the denominator of a fraction.

We can use properties of radicals to help us simplify:

- **Product Property:** $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, when *a* and *b* are positive numbers
- **Quotient Property:** $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, when *a* and *b* are positive numbers

Example 1: Write $\sqrt{72}$ in simplest radical form.

Solution: Find biggest perfect square that goes into 72:

$\sqrt{72} = \sqrt{36}\sqrt{2}$

$= \boxed{6\sqrt{2}}$

OR

Make a factor tree:

$= 2 \cdot 3\sqrt{2}$

$= \boxed{6\sqrt{2}}$

72 → 2 · 36 → 2 · 2 · 18 → 2 · 2 · 2 · 9 → 2 · 2 · 2 · 3 · 3

Circle pairs
- pull pair OUT & multiply
- Anything not circled stays in

Please simplify the following.

a) $\sqrt{75}$

75 → 5 · 15 → 5 · 5 · 3

$\sqrt{25}\sqrt{3}$

$\boxed{5\sqrt{3}}$

b) $\sqrt{48}$

48 → 2 · 24 → 2 · 2 · 12 → 2 · 2 · 2 · 6 → 2 · 2 · 2 · 2 · 3

$\sqrt{16}\sqrt{3}$

$\boxed{4\sqrt{3}}$

$= 2 \cdot 2\sqrt{3}$

$\boxed{4\sqrt{3}}$

c) $\sqrt{120}$

$\sqrt{4}\sqrt{30}$

$\boxed{2\sqrt{30}}$

d) $\sqrt{18}$

$\sqrt{9}\sqrt{2}$

$\boxed{3\sqrt{2}}$

Example 2: Find the product of the following radical expressions and simplify; $(4\sqrt{6})(3\sqrt{10})$

Multiply the numbers outside the radical, multiply the numbers under the radical:

$(4\sqrt{6})(3\sqrt{10}) = 4 \cdot 3\sqrt{6} \cdot \sqrt{10}$

$= 12\sqrt{60}$

$= 12\sqrt{4}\sqrt{15} = 12 \cdot 2\sqrt{15} = \boxed{24\sqrt{15}}$

OR

60 → 2 · 30 → 2 · 2 · 15 → 2 · 2 · 3 · 5

$2 \cdot \sqrt{3 \cdot 5}$

$= 2\sqrt{15}$

$12\sqrt{60}$

$= 12 \cdot 2\sqrt{15}$

$= \boxed{24\sqrt{15}}$

Please simplify the following.

a) $(5\sqrt{3})(2\sqrt{2})$

6 → 2 · 3 ↑ no pairs; radical is in simplest form

$5 \cdot 2\sqrt{3}\sqrt{2}$

$\boxed{10\sqrt{6}}$

b) $(3\sqrt{6})(4\sqrt{2})$

12 → 2 · 6 → 2 · 2 · 3

$2\sqrt{3}$

$3 \cdot 4\sqrt{6}\sqrt{2}$

$12\sqrt{12}$

$12\sqrt{4}\sqrt{3}$

$12 \cdot 2\sqrt{3}$

$\boxed{24\sqrt{3}}$

c) $(5\sqrt{15})(3\sqrt{3})$

$5 \cdot 3\sqrt{15}\sqrt{3}$

$15\sqrt{45}$

$15\sqrt{9}\sqrt{5}$

$15 \cdot 3\sqrt{5}$

$\boxed{45\sqrt{5}}$

45 → 5 · 9 → 5 · 3 · 3

$3\sqrt{5}$

$(\sqrt{\ })^2$ undo each other

Example 3: Square the following radical expression; $\left(3\sqrt{5}\right)^2$

Solution: "Distribute" the power of 2 to each term

$(3\sqrt{5})^2 = 3^2(\sqrt{5})^2$

$= 9 \cdot 5 = \boxed{45}$

Please simplify the following.

a) $\left(\sqrt{7}\right)^2$

$= \boxed{7}$

b) $\left(2\sqrt{3}\right)^2$

$2^2(\sqrt{3})^2$

$= 4 \cdot 3$

$= \boxed{12}$

c) $\left(5\sqrt{2}\right)^2$

$5^2(\sqrt{2})^2$

$= 25 \cdot 2$

$= \boxed{50}$

Example 4: Please use the Quotient Property to simplify this example; $\dfrac{\sqrt{18}}{\sqrt{2}}$

Solution:

Rewrite as a single quotient under radical if possible, then reduce and simplify :

$\dfrac{\sqrt{18}}{\sqrt{2}} = \sqrt{\dfrac{18}{2}} = \sqrt{9} = \boxed{3}$

Please simplify the following.

a) $\sqrt{\dfrac{27}{16}}$ ← other way

27 = 3 · 9 = 3 · 3 · 3 = $3\sqrt{3}$

$\dfrac{\sqrt{27}}{\sqrt{16}} = \boxed{\dfrac{3\sqrt{3}}{4}}$

b) $\dfrac{2\sqrt{6}}{\sqrt{2}}$

$2\sqrt{\dfrac{6}{2}}$

$= \boxed{2\sqrt{3}}$

c) $\dfrac{\sqrt{21}}{\sqrt{3}}$

$\sqrt{\dfrac{21}{3}}$

$= \boxed{\sqrt{7}}$

d) $\dfrac{\sqrt{24}}{\sqrt{6}}$

$\sqrt{\dfrac{24}{6}}$

$= \sqrt{4}$

$= \boxed{2}$

Example 5: Rationalize the denominator for $\dfrac{8}{\sqrt{6}}$

Solution:

"Pop and drop" → Pop the whole square root to the top and drop the sq. root symbol from the bottom

$\dfrac{8}{\sqrt{6}} \uparrow = \dfrac{8\sqrt{6}}{6}$ simplify $\boxed{\dfrac{4\sqrt{6}}{3}}$

Please rationalize the following denominators and simplify.

a) $\dfrac{5}{\sqrt{2}} \uparrow$

$\boxed{\dfrac{5\sqrt{2}}{2}}$

b) $\dfrac{9}{\sqrt{3}} \uparrow$

$\dfrac{9\sqrt{3}}{3} = \boxed{3\sqrt{3}}$

c) $\dfrac{2\sqrt{5}}{\sqrt{7}} \uparrow = \dfrac{2\sqrt{5}\sqrt{7}}{7}$

$= \boxed{\dfrac{2\sqrt{35}}{7}}$