Geometry A
Section 8.3 Notes

Name : Key
Date : ______________ Period : ____

LEARNING TARGETS

- I can use coordinate geometry to identify parallelograms.

You can use the following conditions to determine whether a quadrilateral is a parallelogram.

Conditions for Parallelograms
A quadrilateral is a parallelogram if.... • Both pairs of opposite sides are parallel (definition) • Both pairs of opposite sides are congruent. • Both pairs of opposite angles are congruent. • The diagonals bisect each other. • One pair of opposite sides is congruent and parallel.

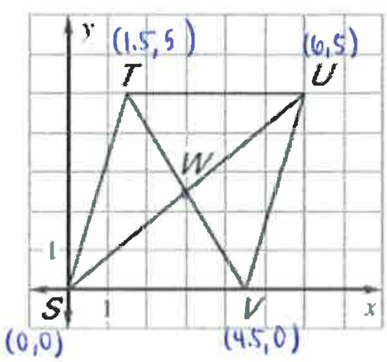
same distances — same slope

Example 1: Find the intersections of diagonals.

diagonals bisect each other so the point where they intersect is the midpoint

The diagonals of $\square STUV$ intersect at point W. Find the coordinates of W.

W(3, 2.5)?



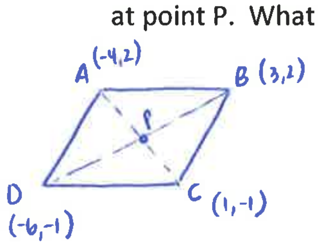
Midpoint of $\overline{SU} = \left(\frac{0+6}{2}, \frac{0+5}{2}\right) = \left(\frac{6}{2}, \frac{5}{2}\right) = (3, 2.5)$

Midpoint of $\overline{TV} = \left(\frac{1.5+4.5}{2}, \frac{5+0}{2}\right) = \left(\frac{6}{2}, \frac{5}{2}\right) = (3, 2.5)$

$\boxed{\text{So W has coordinates } (3, 2.5)}$

Practice:

a. The vertices of $\square ABCD$ are A(-4, 2), B(3, 2), C(1, -1) and D(-6, -1). The diagonals of $\square ABCD$ intersect at point P. What are the coordinates of P?

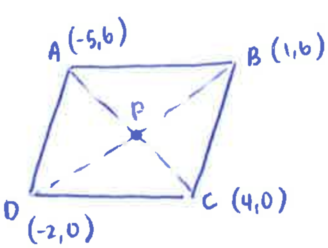


Midpoint of $\overline{AC} = \left(\frac{-4+1}{2}, \frac{2-1}{2}\right) = \left(\frac{-3}{2}, \frac{1}{2}\right) = (-1.5, 0.5)$

Midpoint of $\overline{DB} = \left(\frac{-6+3}{2}, \frac{-1+2}{2}\right) = \left(\frac{-3}{2}, \frac{1}{2}\right) = (-1.5, 0.5)$

$\boxed{\text{So P has coordinates } (-1.5, 0.5)}$

b. The vertices of $\square ABCD$ are A(-5, 6), B(1, 6), C(4, 0) and D(-2, 0). The diagonals of $\square ABCD$ intersect at point P. What are the coordinates of P?



Midpoint of $\overline{AC} = \left(\frac{-5+4}{2}, \frac{6+0}{2}\right) = \left(\frac{-1}{2}, \frac{6}{2}\right) = (-0.5, 3)$

Midpoint of $\overline{DB} = \left(\frac{-2+1}{2}, \frac{0+6}{2}\right) = \left(\frac{-1}{2}, \frac{6}{2}\right) = (-0.5, 3)$

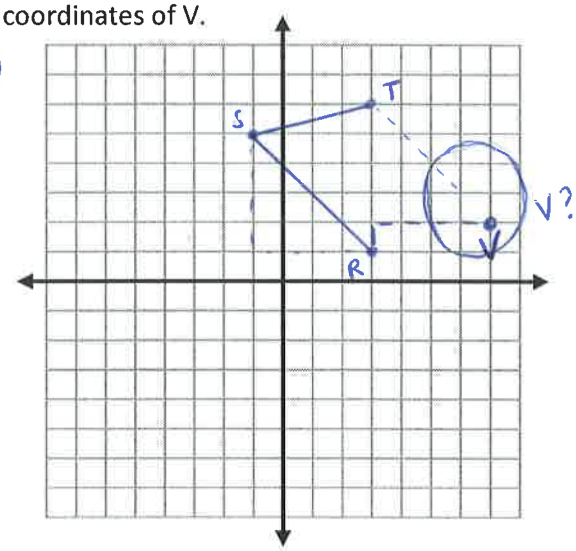
$\boxed{\text{So P has coordinates } (-0.5, 3)}$

Example 2: Find missing coordinate of parallelogram.

If you know the coordinates of three vertices of a parallelogram, you can use slope to find the coordinates of the fourth vertex.

both pairs of opposite sides are parallel

c. Three vertices of ▱*RSTV* are R(3, 1), S(-1, 5), and T(3, 6). Find the coordinates of V.



Find the slope $\left(\frac{\text{rise}}{\text{run}}\right)$ of one of the given segments ($\overline{RS}$ or $\overline{ST}$)

then count that same $\frac{\text{rise}}{\text{run}}$ on its opposite side

ST : slope = $\frac{\text{rise}}{\text{run}} = \frac{1}{4}$ so count $\frac{\text{rise}}{\text{run}}$ of $\frac{1}{4}$ from R

$\boxed{\text{OR}}$

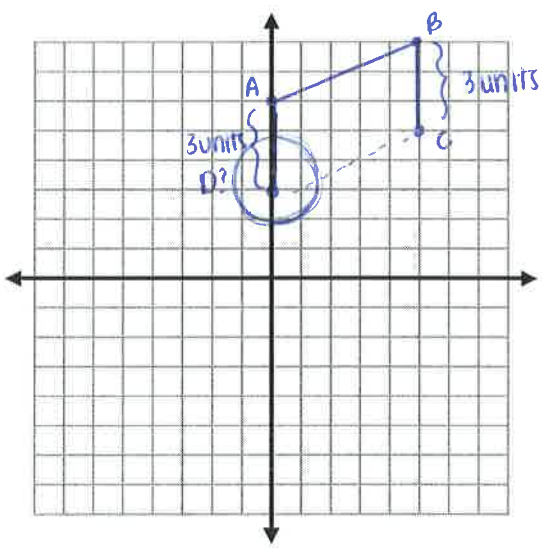
SR : slope = $\frac{\text{rise}}{\text{run}} = \frac{-4}{4}$ so count $\frac{\text{rise}}{\text{run}}$ of $\frac{-4}{4}$ from T

Practice:

$\boxed{\text{so V has coordinates (7,2)}}$

The coordinates of three vertices of a parallelogram are given. Find the coordinates of the fourth vertex.

d. ▱*ABCD* with A(0,6), B(5, 8), and C(5, 5)



For this one, we dont even need to count our $\frac{\text{rise}}{\text{run}}$! Since we know opposite sides in a parallelogram are congruent and we can count 3 units from B to C, we can do the same on the opposite side from A!

So point D would be 3 units down from A at point $\boxed{(0,3)}$.

Example 3: Use coordinate Geometry to identify parallelograms

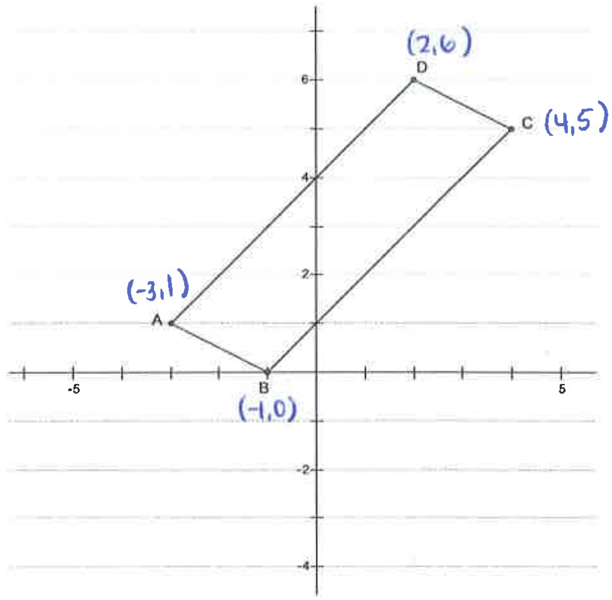
e. Show that *ABCD* is a parallelogram. by showing one pair of opp sides is both parallel and congruent

Congruence:

$AB = \sqrt{(-3-\text{-}1)^2+(1-0)^2} = \sqrt{(-3+1)^2+(1)^2} = \sqrt{(-2)^2+(1)^2} = \sqrt{4+1} = \sqrt{5}$

$DC = \sqrt{(2-4)^2+(6-5)^2} = \sqrt{(-2)^2+(1)^2} = \sqrt{4+1} = \sqrt{5}$

$\overline{AB} \cong \overline{DC}$



Parallel:

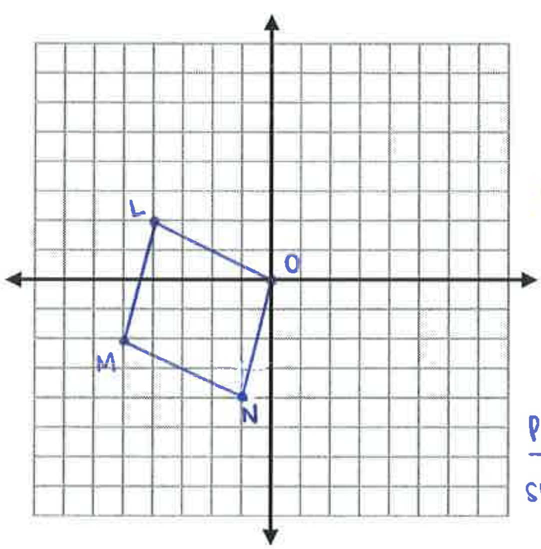
Slope of $\overline{AB} = \frac{1-0}{-3-\text{-}1} = \frac{1-0}{-3+1} = \frac{1}{-2}$

Slope of $\overline{DC} = \frac{6-5}{2-4} = \frac{1}{-2}$

$\overline{AB} \parallel \overline{DC}$

Since $\overline{AB} \cong \overline{DC}$ and $\overline{AB} \parallel \overline{DC}$, ABCD is a parallelogram

f. The vertices of LMNO are L(-4, 2), M(-5, -2), N(-1, -4) and O(0, 0). Show that LMNO is a parallelogram.



show that sides $\overline{LO}$ and $\overline{MN}$ are both congruent and parallel (or $\overline{LM}$ and $\overline{ON}$)

Congruence:

$LO = \sqrt{(-4-0)^2+(2-0)^2} = \sqrt{(-4)^2+(2)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

$MN = \sqrt{(-5-\text{-}1)^2+(-2-\text{-}4)^2} = \sqrt{(-5+1)^2+(-2+4)^2} = \sqrt{(-4)^2+(2)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

Parallel:

slope of $\overline{LO} = \frac{2-0}{-4-0} = \frac{2}{-4} = -\frac{1}{2}$

slope of $\overline{MN} = \frac{-2-\text{-}4}{-5-\text{-}1} = \frac{-2+4}{-5+1} = \frac{2}{-4} = -\frac{1}{2}$

since $\overline{LO} \cong \overline{MN}$ and $\overline{LO} \parallel \overline{MN}$, LMNO is a parallelogram