



- I can find angle measures in polygons.
- I can use properties of parallelograms to find side lengths and angle measures.
- I can use properties to identify parallelograms.
- I can use coordinate geometry to identify parallelograms.

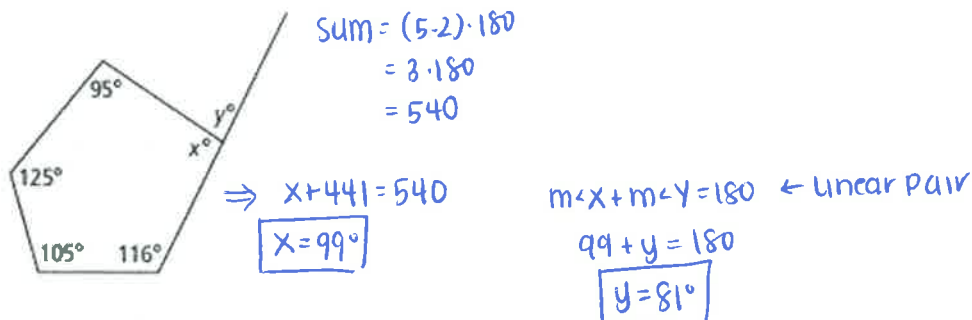
I can find angle measures in polygons.

- ✓ I can find the sum of interior angles of a polygon using  $(n-2) \cdot 180$
- ✓ I can find the measure of each interior angle of a regular polygon using  $\frac{(n-2) \cdot 180}{n}$
- ✓ I can find the sum of the exterior angles of a polygon using 360
- ✓ I can find the measure of each exterior angle of a polygon using  $\frac{360}{n}$

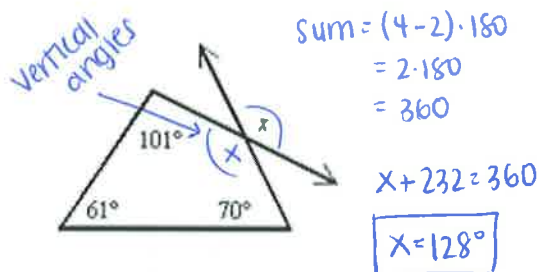
1) What is the sum of the interior angle measures of a 15-gon?  $n=15$

$$\begin{aligned} \text{sum} &= (15-2) \cdot 180 \\ &= 13 \cdot 180 = \boxed{2340^\circ} \end{aligned}$$

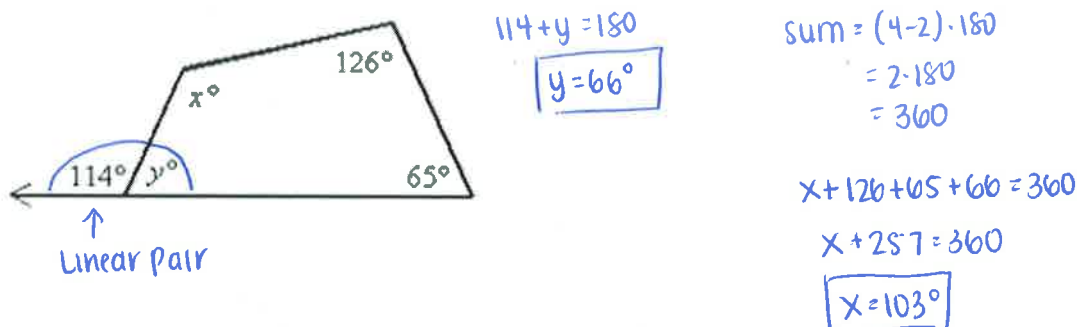
2) Find the values of the variables in the figure below.



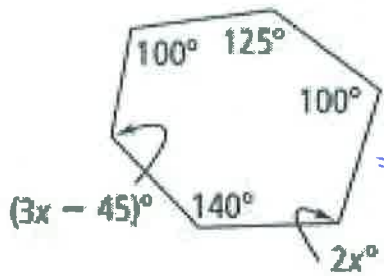
3) Find the value of x.



4) Find the values of x and y.



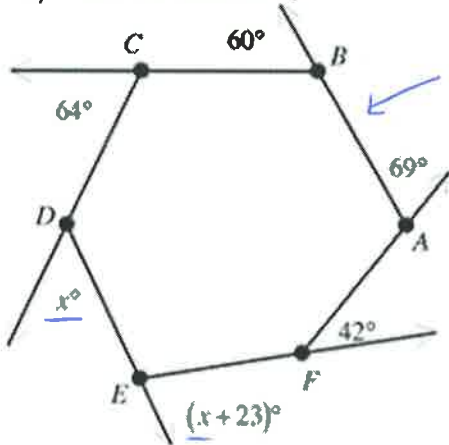
5) Find the value of  $x$ .



$$\begin{aligned} \text{Sum} &= (6-2) \cdot 180 \\ &= 4 \cdot 180 \\ &= 720 \end{aligned}$$

$$\begin{aligned} \Rightarrow 5x + 420 &= 720 \\ 5x &= 300 \\ \boxed{x} &= \boxed{60} \end{aligned}$$

6) Find the value of  $x$ .



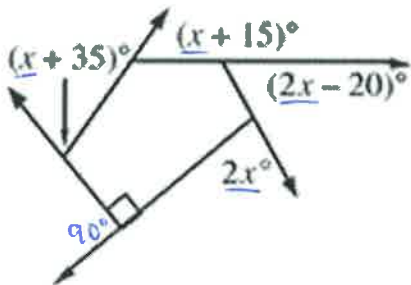
exterior angles sum to  $360^\circ$

$$2x + 258 = 360$$

$$2x = 102$$

$$\boxed{x} = \boxed{51}$$

7) Find the value of  $x$ .



$$6x + 120 = 360$$

$$6x = 240$$

$$\boxed{x} = \boxed{40}$$

8) What is the measure of each interior and each exterior angle of a regular decagon?  $n=10$

$$\text{Interior} = \frac{(10-2) \cdot 180}{10}$$

$$= \frac{8 \cdot 180}{10}$$

$$= \frac{1440}{10} = \boxed{144^\circ}$$

$$\text{Exterior} = \frac{360}{n}$$

$$= \frac{360}{10}$$

$$= \boxed{36^\circ}$$

9) Find the number of sides of a convex polygon if the sum of the measures of its interior angles is  $2880^\circ$ .

$$\text{Sum} = (n-2) \cdot 180$$

$$\frac{2880}{180} = \frac{(n-2) \cdot 180}{180}$$

$$16 = n - 2$$

$$\boxed{n} = \boxed{18}$$

10) Find the number of sides of a regular polygon with each interior angle equal to  $171^\circ$ .

$$\frac{171}{1} = \frac{(n-2) \cdot 180}{n}$$

$$171n = (n-2) \cdot 180$$

$$171n = 180n - 360$$

$$-9n = -360$$

$$n = 40$$

11) You are designing patterns for your art project. Can you form regular polygons with the given interior angle measures? If yes, how many sides would the regular polygon have?

a)  $\frac{135}{1} = \frac{(n-2) \cdot 180}{n}$

$$135n = (n-2) \cdot 180$$

$$135n = 180n - 360$$

$$-45n = -360$$

$$n = 8 \leftarrow \text{possible}$$

b)  $\frac{130}{1} = \frac{(n-2) \cdot 180}{n}$

$$130n = (n-2) \cdot 180$$

$$130n = 180n - 360$$

$$-50n = -360$$

$$n = 7.2 \leftarrow \text{not possible, can't have a polygon with 7.2 sides}$$

I can use properties of parallelograms to find side lengths and angle measures.

- ✓ I can find lengths of opposite sides on parallelograms.
- ✓ I can find measures of opposite angles in parallelograms.
- ✓ I can find measures of consecutive angles in parallelograms.
- ✓ I can find lengths of diagonals in parallelograms.
- ✓ I can use properties of parallel lines to find angle measures in parallelograms.

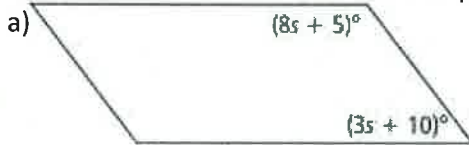
12) Consecutive angles in a parallelogram are always \_\_\_\_\_.

- A) Congruent angles.
- B) Complementary angles.
- C) Supplementary angles.
- D) Vertical angles.

13) Choose the statement that is NOT ALWAYS true. For any parallelogram \_\_\_\_\_.

- A) The diagonals bisect each other.
- B) Opposite angles are congruent.
- C) The diagonals are perpendicular.
- D) Opposite sides are congruent.

14) Find the value of the variable in each parallelogram below. Justify your reasoning.

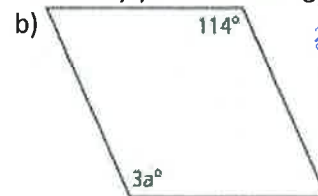


Consecutive angles are supplementary

$$8s + 5 + 3s + 10 = 180$$

$$11s + 15 = 180$$

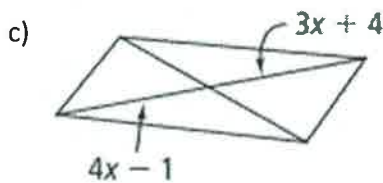
$$11s = 165 \Rightarrow s = 15$$



$$3a = 114$$

$$a = 38$$

opposite angles are congruent



$$4x - 1 = 3x + 4$$

$$x = 5$$

diagonals bisect each other



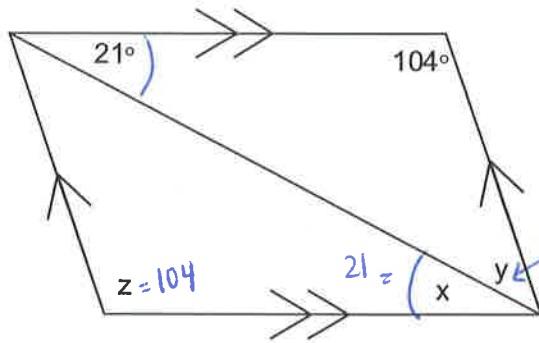
opposite sides are congruent

$$3x - 2 = 19$$

$$3x = 21$$

$$x = 7$$

15) Find the values of the variables in the parallelogram.



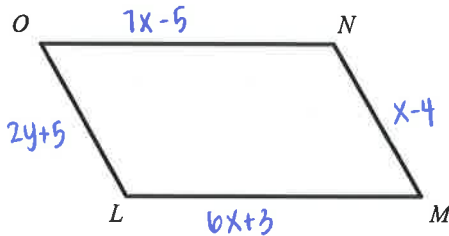
$z = 104$  ← opp angles are  $\cong$

$x = 21^\circ$  ← alternate interior angles

$180 - 104 - 21 = 55^\circ$  ← triangle sum

$y = 55$

16) If  $ON = 7x - 5$ ,  $LM = 6x + 3$ ,  $NM = x - 4$ , and  $OL = 2y + 5$ , find the values of  $x$  and  $y$  given that  $LMNO$  is a parallelogram.



$7x - 5 = 6x + 3$   
 $x = 8$

$2y + 5 = x - 4$

$2y + 5 = 8 - 4$

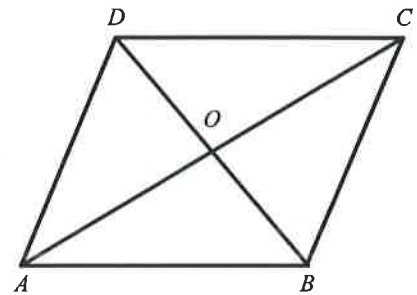
$2y + 5 = 4$

$2y = -1$

$y = -\frac{1}{2}$

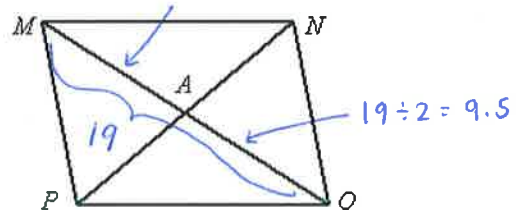
17) Complete each statement for parallelogram  $ABCD$ . Then justify your answer.

- A)  $\overline{AD} \cong \underline{\overline{BC}}$  ← opp sides are  $\cong$
- B)  $\overline{OC} \cong \underline{\overline{OA}}$  ← diagonals bisect each other
- C)  $\overline{CD} \parallel \underline{\overline{AB}}$  ← opp sides are parallel
- D)  $\angle CBA \cong \underline{\angle ADC}$  ← opp angles are  $\cong$



18) Find  $AM$  in the parallelogram if  $PN = 10$  and  $MO = 19$ .

$AM = 9.5$

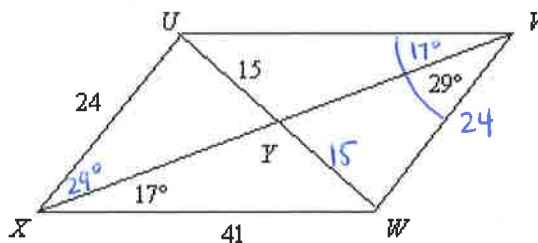


$19 \div 2 = 9.5$

$19 \div 2 = 9.5$

19) Use the diagram below to find each of the following.

- a)  $m\angle WVU = \underline{17 + 29 = 46^\circ}$
- b)  $WV = \underline{24}$
- c)  $m\angle XUV = \underline{134^\circ}$
- d)  $UW = \underline{15 + 15 = 30}$



$46 + m\angle XUV = 180$   
 $m\angle XUV = 134^\circ$

20) In the diagram below, please find AC and BD.

$$AC = (7-2) + 2(7) - 9$$

$$AC = 5 + 5$$

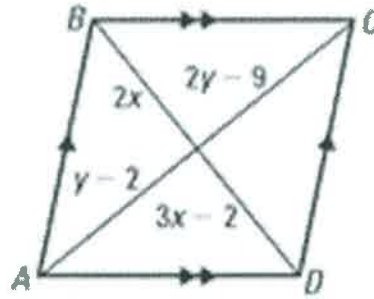
$$AC = 10$$

$$BD = 2(2) + 3(2) - 2$$

$$= 4 + 6 - 2$$

$$= 4 + 4$$

$$BD = 8$$



$$2x = 3x - 2$$

$$-1x = -2$$

$$x = 2$$

$$2y - 9 = y - 2$$

$$y - 9 = -2$$

$$y = 7$$

□ I can use properties to identify parallelograms.

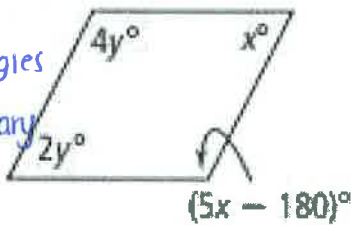
21) In each of the following, for what values of x and y must each figure be a parallelogram. Explain.

a)

$2y + 4y = 180$  ← consec. angles are supplementary

$$6y = 180$$

$$y = 30$$



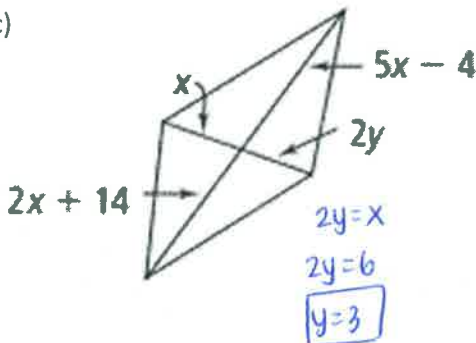
$x + 5x - 180 = 180$  ← consec angles are supplementary

$$6x - 180 = 180$$

$$6x = 360$$

$$x = 60$$

c)



$$5x - 4 = 2x + 14$$

$$3x - 4 = 14$$

$$3x = 18$$

$$x = 6$$

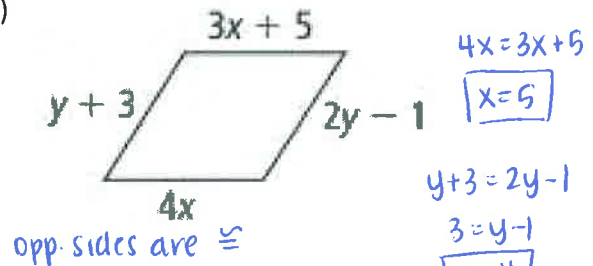
$$2y = x$$

$$2y = 6$$

$$y = 3$$

diagonals bisect each other

b)



$$4x = 3x + 5$$

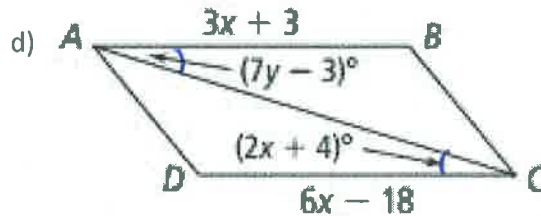
$$x = 5$$

$$y + 3 = 2y - 1$$

$$3 = y - 1$$

$$y = 4$$

opp. sides are  $\cong$



$3x + 3 = 6x - 18$  ← opp sides are  $\cong$

$$3 = 3x - 18$$

$$21 = 3x$$

$$x = 7$$

$7y - 3 = 2x + 4$  ← alt. int. angles are  $\cong$

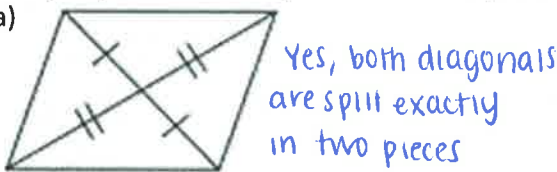
$$7y - 3 = 2(7) + 4$$

$$7y - 3 = 18$$

$$7y = 21 \Rightarrow y = 3$$

22) Can you prove that the quadrilateral is a parallelogram based on the given information? Explain.

a)



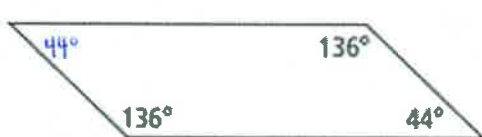
Yes, both diagonals are split exactly in two pieces

b)



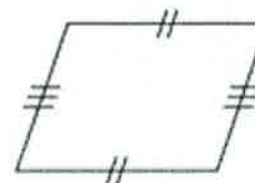
No, not enough information

c)



Yes, both pairs of opposite angles are congruent

d)



Yes, both pairs of opposite sides are congruent

□ I can use coordinate geometry to identify parallelograms in the coordinate plane.

23) Given: Quadrilateral ABCD with A(-5, 0), B(4, -3), C(8, -1) and D(-1, 2).

Prove: ABCD is a parallelogram using the slope formula. ← both pairs of opp sides are parallel

$$\text{slope}_{\overline{AD}} = \frac{2-0}{-1-5} = \frac{2-0}{-1+5} = \frac{2}{4} = \frac{1}{2}$$

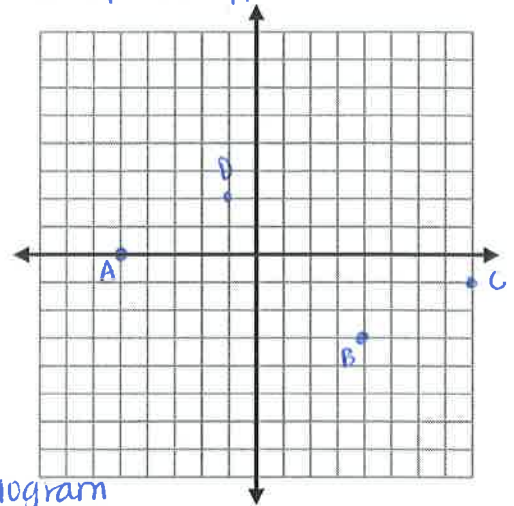
$$\text{slope}_{\overline{BC}} = \frac{-1-3}{8-4} = \frac{-1-3}{4} = \frac{2}{4} = \frac{1}{2}$$

}  $\overline{AD} \parallel \overline{BC}$

$$\text{slope}_{\overline{AB}} = \frac{-3-0}{4-5} = \frac{-3}{4+5} = \frac{-3}{9} = -\frac{1}{3}$$

$$\text{slope}_{\overline{CD}} = \frac{2-1}{-1-8} = \frac{2+1}{-1-8} = \frac{3}{-9} = -\frac{1}{3}$$

}  $\overline{AB} \parallel \overline{CD}$



\* Since both pairs of opposite sides are parallel, ABCD is a parallelogram

24) Given: Quadrilateral ABCD with A(-5, -5), B(-2, 4), C(6, 4) and D(3, -5)

Prove: ABCD is a parallelogram using the distance formula. ← both pairs of opposite sides are congruent

$$AB = \sqrt{(-2-5)^2 + (4-5)^2} = \sqrt{(-2+5)^2 + (4+5)^2} = \sqrt{(3)^2 + (9)^2} = \sqrt{90}$$

$$CD = \sqrt{(3-6)^2 + (-5-4)^2} = \sqrt{(-3)^2 + (-9)^2} = \sqrt{9+81} = \sqrt{90}$$

$$\overline{AB} \cong \overline{CD}$$

Since  $\overline{BC}$  and  $\overline{AD}$  are perfectly horizontal, we can count units instead of using the distance formula!

$$BC = 8$$

$$AD = 8$$

}  $\overline{BC} \cong \overline{AD}$

\* Since both pairs of opposite sides are congruent, ABCD is a parallelogram

25) Given: Quadrilateral ABCD with A(-2, 3), B(3, 2), C(2, -1) and D(-3, 0)

Prove: ABCD is a parallelogram using the midpoint formula. ← both diagonals have the same midpoint

$$\text{Midpoint of diagonal } \overline{AC} = \left( \frac{-2+2}{2}, \frac{3+(-1)}{2} \right) = \left( \frac{0}{2}, \frac{2}{2} \right) = (0, 1)$$

$$\text{Midpoint of diagonal } \overline{BD} = \left( \frac{-3+3}{2}, \frac{0+2}{2} \right) = \left( \frac{0}{2}, \frac{2}{2} \right) = (0, 1)$$

\* Since the midpoint of each diagonal is (0, 1) then ABCD is a parallelogram

