Geometry A
6.4: AA~ Applications

Name: key
Date: _____ Period: ____

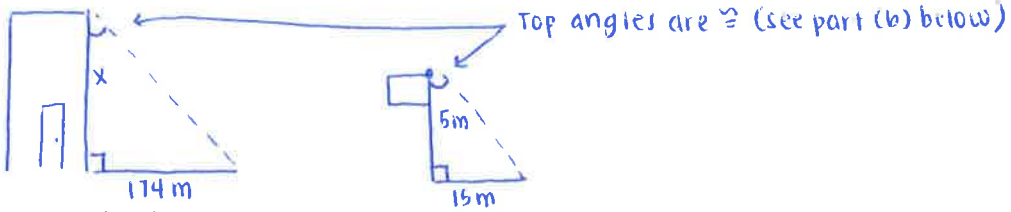
- I can use AA~ to set up and solve proportions.
- I can use AA~ to solve indirect measurement problems in the real world.

The heights of very tall structures or structures that would be impossible to measure in a traditional manner can be measured indirectly using similar figures and proportions. This method is called ***indirect measurement.***

Example 1: Shadow reckoning
This method is credited to Thales, a Greek scientist, engineer, and mathematician. Legend says that he was the first to determine the height of the pyramids in Egypt by examining shadows made by the sun. Here is an example of this technique:

A building casts a shadow 174 meters long. At the same time, a pole 5 meters high casts a shadow 15 meters long. What is the height of the building?

a) Draw and label a diagram to represent the scenario described.



b) Since the shadows were formed at the same time, the angles formed by the shadow and the sun's ray are congruent. What other angles can we say are congruent?

The right angle made with the building & ground as well as the pole & ground.

c) Are the triangles similar? If so, can we set up a proportion to find the height of the building? What is the height of the building?

Yes, the triangles are similar by AA~

$\frac{\text{Building}}{\text{Pole}}$ $\frac{x}{5} = \frac{174}{15} \Rightarrow 15x = 870 \Rightarrow x = 58$

(x: height, 174: shadow)

The height of the building is 58 m.

Example 2: Shadow technique with a twist

A building height is 30 ft. A man is standing on the building of height *h* ft. The shadow of the building is 40 ft and shadow of man is 10 ft. So find the height of man as shown in the figure.

$\frac{\text{Small } \Delta}{\text{big } \Delta}$: $\frac{30}{30+h} = \frac{40}{50} \Rightarrow 1500 = 40(30+h)$

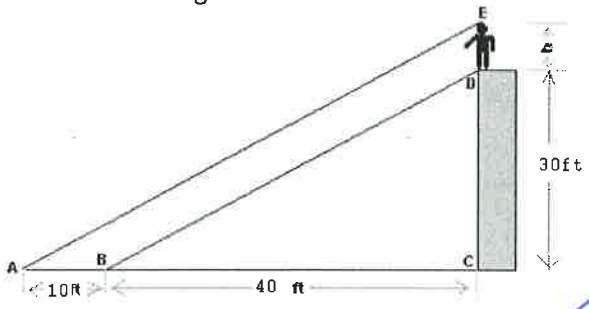
(30, 30+h: heights; 40, 50: ground lengths)

$1500 = 1200 + 40h$

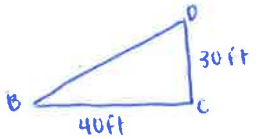
$300 = 40h$

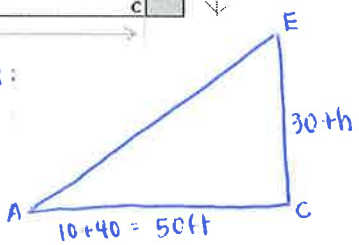
$h = 7.5$

The man is 7.5 ft tall



Separate the 2 triangles:





Example 3: Using a mirror

We can place a mirror on the ground to help us estimate the height of tall objects. Here is an example of this technique:

Raymond wanted to estimate the height of a tall tree. He placed a mirror on the ground 26 feet from the tree, and walked backwards until he could see the top of the tree in the center of the mirror. Raymond is now 6.5 feet from the mirror, and his eye level is about 5.5 feet above the ground. What is the height of the tree?

a) Draw a diagram to represent the scenario.

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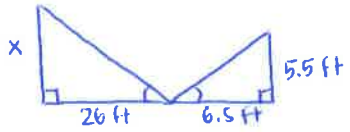


b) Since the angle of reflection is equal to the angle of refraction, the two angles at the mirror are congruent. What other angles can we state are congruent?

The angle made at the base of the tree and the ground is a right angle and so is the angle made by Raymond's feet and the ground.

c) Are the triangles similar? If so, can we set up a proportion to find the height of the tree? What is the height of the tree?

Yes, the triangles are similar by AA ~

$\frac{\text{tree}}{\text{Ray}}: \frac{x}{5.5} = \frac{26}{6.5} \Rightarrow 6.5x = 143$

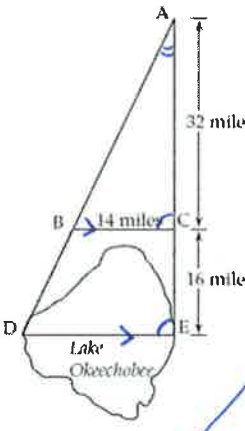
(x: height; 26: base)

$x = 22$

The tree is 22 ft tall

Example 4: Distance across a lake

A surveyor used the map to the right to find the distance across Lake Okeechobee. In the diagram, $\overline{BC} \parallel \overline{DE}$.



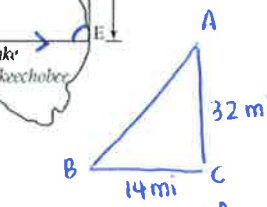
a) Explain why $\Delta ABC \sim \Delta ADE$

Both triangles share $\angle A$; so $\angle A \cong \angle A$ by Reflexive Prop.

Since $\overline{BC} \parallel \overline{DE}$, $\angle ACB \cong \angle AED$ by corresponding Angles Postulate

So $\Delta ABC \sim \Delta ADE$ by AA ~

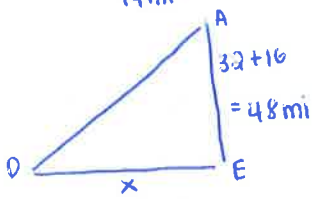
b) Set up and solve a proportion to find the length of Lake Okeechobee.



$\frac{\text{small } \Delta}{\text{big } \Delta}: \frac{32}{48} = \frac{14}{x} \Rightarrow 32x = 672 \Rightarrow x = 21$

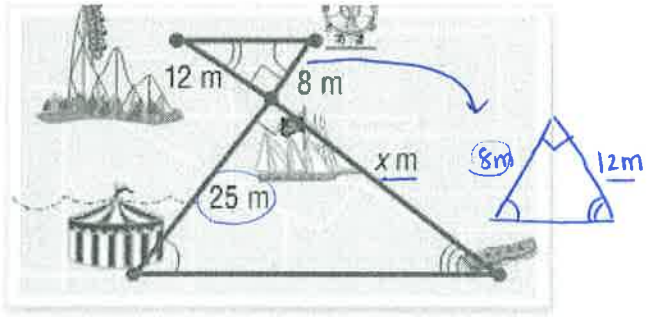
(32, 48: heights; 14, x: bases)

Lake Okeechobee is 21 miles long



Example 5:

How far is it from the log ride to the pirate ship?



$\frac{\text{big } \Delta}{\text{sm } \Delta}: \frac{25m}{8m} = \frac{x\,m}{12m}$

$\Rightarrow 8x = 300$

$x = 37.5$

It is 37.5 m from the log ride to the pirate ship