Geometry A
4.4: Prove Triangles Congruent by H-L

Name: $\qquad$
Date: $\qquad$ Period: $\qquad$

- I can prove triangles congruent using H-L


## Vocabulary:

In a right triangle, the side opposite the right angle is called the $\qquad$ -

In a right triangle, the sides that form the right angle are called the $\qquad$ .

In right triangle $A B C$, the hypotenuse is $\qquad$ .

The legs are $\qquad$ and $\qquad$ .


There is a special method for proving right triangles are congruent. This method only works for right triangles!

| Hypotenuse - Leg Theorem (H-L) | Example: |
| :--- | :--- |
| If the hypotenuse and leg of one right triangle are |  |
| congruent to the hypotenuse and leg of a second <br> right triangle, then the two triangles are congruent. | If Hypotenuse $\overline{B C} \cong$ <br> and Leg $\overline{A B} \cong \ldots$ <br> $\triangle A B C$ and $\triangle D E F$, in right triangles <br> $\triangle A B C \cong \ldots$ |

## Example 1: Using H-L to identify congruent triangles

Can you prove the following triangles are congruent? Explain.
a.

b.


C.

d.


When writing a proof using H-L, it is important that you state the following three things in your explanation:

- That the two triangles are right triangles.
- One pair of legs is congruent.
- The two hypotenuse are congruent.


## Example 2: Proofs involving H-L

a) Given: $\overline{A C} \cong \overline{E C} ; \overline{A B} \perp \overline{B D} ; \overline{E D} \perp \overline{B D}$;
$\overline{A C}$ is a bisector of $\overline{B D}$
Prove: $\triangle A B C \cong \triangle E D C$


| Statements | Reasons |
| :--- | :--- |
| 1. $\overline{A C} \cong \overline{E C}$ | 1. |
| 2. $\overline{A B} \perp \overline{B D} ; \overline{E D} \perp \overline{B D}$ | 2. |
| 3. | 3. |
| 4. | 4. |
| 5. $\overline{A C}$ is a bisector of $\overline{B D}$ | 5. |
| 6. $\overline{B C} \cong$ | 6. |
| 7. $\triangle A B C \cong \triangle E D C$ | 7. |

b) Given: $\overline{A B} \cong \overline{D C} ; \overline{B A} \perp \overline{A C} ; \overline{C D} \perp \overline{D B}$ Prove: $\triangle A B C \cong \triangle D C B$


| Statements | Reasons |
| :--- | :--- |
| 1. $\overline{A B} \cong \overline{D C}$ | 2. |
| 2. $\overline{B A} \perp \overline{A C} ; \overline{C D} \perp \overline{D B}$ | 3. |
| 3. $\angle A$ and $\angle D$ are | 4. |
| 4. $\overline{\triangle A B C \text { and } \triangle E D C \text { are }}$ | 5. |
| 5. $\overline{\overline{C B} \cong \overline{C B}}$ | 6. |
| 6. $\triangle A B C \cong \triangle D C B$ |  |

Does a right angle always mean we will use H-L? Let's see!

Given: $\overline{A B}$ is perpendicular bisector of $\overline{C D}$
Prove: $\triangle A B C \cong \triangle A B D$


| Statements | Reasons |
| :--- | :--- |
| 1. $\overline{A B}$ is perpendicular bisector of $\overline{C D}$ | 1. |
| 2. | 2. definition of perpendicular lines |
| 3. $\angle A B C \cong \angle A B D$ | 3. |
| 4. $\overline{B C} \cong$ | 4. |
| 5. $\overline{A B} \cong \overline{A B}$ | 5. |
| 6. $\triangle A B C \cong \triangle A B D$ | 6. |

$\qquad$
$\qquad$ Period: $\qquad$

Is it possible to prove that the triangles are congruent? If so, state the postulate or theorem you would use.
1.

2.


3.

4.

5.

6.


State the third congruence that is needed to prove that $\triangle A B C \cong \triangle X Y Z$ using the given postulate or theorem.
7. GIVEN: $\angle B \cong \angle E, \overline{B C} \cong \overline{E F}$, $\qquad$ $\cong$ $\qquad$
Use the SAS Congruence Theorem
8. GIVEN: $\overline{A B} \cong \overline{D E}, \overline{B C} \cong \overline{E F}$, $\qquad$ $\cong$ $\qquad$
Use the SSS Congruence Postulate

9. GIVEN: $\overline{A C} \cong \overline{D F}, \angle A$ is a right angle and
$\angle A \cong \angle D$, $\qquad$ $\cong$ $\qquad$
Use the H-L Congruence Theorem
10. Complete the proof.

Given: $\overline{O M} \perp \overline{L N}, \overline{M L} \cong \overline{M N}$
Prove: $\triangle O M L \cong \triangle O M N$


| Statements | Reasons |
| :--- | :--- |
| $1 \overline{O M} \perp \overline{L N}$ | 1. |
| $2 . \angle L M O$ and $\angle N M O$ are right angles | $2 . \quad$ def. of |
| $3 . \quad \triangle L M O$ and $\triangle N M O$ are right triangles | $3 . \quad$ Def. of |
| $4 . \overline{M L} \cong \overline{M N}$ | 4. |
| 5. | 5. Reflexive Property |
| $3 . \quad \triangle O M L \cong \triangle O M N$ | 6. |

11. Given: $\angle \mathrm{JKL} \& \angle \mathrm{MLK}$ are right angles $\overline{J L} \cong \overline{M K}$
Prove: $\triangle N K \cong \triangle M L K$


| Statements | Reasons |
| :--- | :--- |
| 1. $\quad \triangle J K L \& \angle M L K$ are right $\angle \mathrm{s}$ | 1. |
| 2. $\quad \triangle K L$ and $\triangle M L K$ are | 2. |
| 3. $\overline{\overline{J L} \cong \overline{M K}}$ | 3. |
| 4. $\overline{K L} \cong \overline{K L}$ | 4. |
| 5. $\quad \Delta K L \cong \triangle M L K$ | 5. |

12. Given: $\overline{A B} \cong \overline{D B}, \overline{B C} \perp \overline{A D}$

Prove: $\triangle A B C \cong \triangle D B C$


| Statements | Reasons |
| :--- | :--- |
| 1. $\overline{A B} \cong \overline{D B}$ | 1. |
| 2. $\overline{B C} \perp \overline{A D}$ | 2. |
| 3. $\angle B C A$ and $\angle B C D$ are | 4. |
| 4. $\overline{\triangle A B C \text { and } \triangle D B C \text { are }}$ | 5. |
| 5. $\overline{\overline{C B} \cong \overline{C B}}$ | 6. |

## Answer Key:

1. Yes, by SAS
2. Yes, by AAS
3. Yes, by SAS
4. Yes, by SAS
5. Yes, by SAS
6. Yes, by H-L
7. $\overline{B A} \cong \overline{E D}$
8. $\overline{A C} \cong \overline{D F}$
9. $\overline{B C} \cong \overline{E F}$
10. 11) Given 2) Def of Perpendicular Lines 3) Def of right triangles 4) $\overline{O M} \cong \overline{O M}$ 5) SAS
1. 2) Given 2) Right triangles; Def of right triangles 3) Given 4) Reflexive Property 5) H-L
1. 2) Given 2) Given 3) Right angles; Def of perpendicular lines 4) Right triangles; Def of right triangles 5) Reflexive Property 6) H-L
